**EXERCISES OF MATRICES OPERATIONS - SOLUTIONS**

Throughout, we assume that the dimensions of the matrices in this note make sense.

**Question 1.** Which of the following statements must be true?

(1) F. Counterexample: $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(2) F.

(3) F. Counterexample: $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(4) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(5) F. Counterexample: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(6) F. Counterexample: $A$ is the $n \times n$ with 1 on the $(n-1)$ entries to the right of the main diagonal and zero elsewhere.

(7) T.

(8) F. $-A$ is still symmetric.

(9) F.

(10) T.

(11) F. Counterexample: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(12) F.

(13) T.

(14) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(15) T.

(16) F. Counterexample: $A$ is a $3 \times 5$ matrix with rank 2.

(17) F. Counterexample: $A = I_n$ and $B = -I_n$ .

(18) T.

(19) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(20) T.

(21) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(22) F. Counterexample: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(23) F. Counterexample: $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(24) T.

(25) T.

(26) T.

(27) T.

**Question 2.** If $A$ is row equivalent to $B$, then which of the following statements must be true?

(1) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(2) T.

(3) T.

(4) F. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(5) T.

(6) T.

(7) T.